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PREDICTION OF HEAT TRANSFER COEFFICIENT IN TURBULENT SLURRY FLOW WITH ENHANCED DAMPING OF TURBULENCE

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The paper deals with heat transfer in fully developed turbulent pipe flow of fine-dispersive slurry, which exhibits damping of turbulence. The slurry, which is homogeneous, contains fine-particles and its concentration varies from 0% to 30% by volume. As such slurry exhibits a yield stress the Bingham model was chosen in order to calculate apparent viscosity. The main objective of the paper is to examine the influence of solids concentration on heat transfer coefficient by taking into account a turbulence damping function especially designed for such slurry. The mathematical model constitutes time averaged momentum and energy differential equations, and a two-equation turbulence model. The mathematical model is able to predict velocity distribution, frictional head loss, temperature distribution and Nusselt number of fine-dispersive slurry with a yield stress. Results of numerical predictions demonstrate the importance of solids concentration influence on heat transfer coefficient and are presented as figures and conclusions.

Key words: slurry flow, modelling of turbulence, heat transfer in slurry flow.

NOTATION

C_i	– constant in Launder and Sharma turbulence model, i=1, 2
C_V	– solids concentration (volume fraction of solids averaged in cross section) %
c_P	– specific heat at constant pressure, J/(kg K)
D	– inner pipe diameter, m
f_μ	– turbulence damping function at the pipe wall
k	– kinetic energy of turbulence, m^2/s^2
Pr	– Prandtl number
p	– static pressure, Pa
q	– input power of heat per unit pipe length, W/m
r	– distance from symmetry axis, m
R	– inner pipe radius, m
Re	– Reynolds number
T	– temperature, K
U	– velocity component in ox direction, m/s
u', v'	– fluctuating components of velocity U and V, m/s
x	– coordinate for ox direction, m
$\overline{\ }$	– time averaged

GREEK SYMBOLS

α – heat transfer coefficient, W/(m^2 K)
λ – thermal conductivity, W/(m K)
ε – rate of dissipation of kinetic energy of turbulence, m^2/s^3
μ – viscosity, Pa·s
μ_{PL} – plastic viscosity in Bingham rheological model, Pa·s
ν – kinematic viscosity coefficient, m^2/s
ρ – density, kg/m^3
σ_i – diffusion coefficients in k–ε turbulence model, i = k, ε
τ – shear stress, Pa
τ_o – yield shear stress, Pa

INDEXES

ap – apparent
b – bulk (cross-section averaged value)
i – index, i = 1, 2
m – mixture (solid-liquid)
t – turbulent
w – solid wall

1. INTRODUCTION

Solid-liquid flow, named as slurry flow, appears frequently in chemical engineering, power plants, food and mining industries and is often strongly influenced by heat exchange between the transported materials and the surrounding (Rozenblit et al., 2000). Solid-liquid flows are classified as settling or non-settling types. Settling slurries are formed mainly by coarse particles (Kaushal and Tomita, 2007; Matousek and Krupicka, 2013). However, settling can also exist in slurries with medium or fine solid particles for sufficiently low bulk velocity. However, when predicting the frictional head loss of settling slurry flow with coarse or medium solid particles, it is reasonable to assume the Newtonian model, as now one can measure rheology in such settling slurries (Shook and Roco, 1991).

Non–settling slurries contain fine particles and can form a stable homogeneous mixture exhibiting increased apparent viscosity. Such slurries usually exhibit a yield stress and require an adequate rheological model. Usually they demonstrate a thicker viscous sublayer, resulting in increased damping of turbulence near a pipe wall. The phenomenon of the thickness of the viscous sublayer was reported by some of researchers, including Wilson and Thomas, whose contribution is essential (Wilson and Thomas, 1985). The mathematical model for fine-dispersive slurry flow for isothermal and non-isothermal flow conditions should include an apparent viscosity concept, with the support of an adequate rheological model, and time averaged momentum and energy equations. When using the turbulence model in order to calculate the turbulent stress tensor in the momentum equation, a properly defined wall damping function is also required (Bartosik, 2009, 2010).

There are several turbulence models dedicated to Newtonian slurry flows, for instance: one-equation turbulence models of Mishra et al. (1998) or two-equation model of Launder and Sharma (1974), or k-ε-Ap model of Yulin (1996). Mishra et al. (1998) built and improved one-equation 'k-l' turbulence model using an empirical turbulence length scale. The two-equation k-ε-A_p turbulence model of Yulin (1996) is built using the kinetic energy of turbulence and its dissipation rate as in the standard turbulence model for a single-phase flow. The 'A_p' is an algebraic equation describing the solid phase. This mathematical model has been successfully examined, however, only for low solids concentration.

Stainsby and Chilton (1996) developed a hybrid model for non-Newtonian slurries in which the apparent viscosity was calculated by the Herschel-Bulkley rheological model at a low strain rate and by the Bingham model at a high strain rate. Using the time-averaged momentum equation and the k-ε turbulence model of Launder and Sharma (1974), recommended previously by Bartosik and Shook (1991), they were able to predict frictional head loss and velocity distributions in fine-dispersive slurry flow. They did not include any changes in the k-ε turbulence model. Their hybrid model has been successfully examined only for low solids concentration and low yield stresses, and for maximum slurry density equal to 1105 kg/m^3.

There are several researchers who simulated slurry flow using different approaches, like for instance Gupta and Pagalthivarthi (2009), Ravikumar et al. (2013), Talmon (2013), Silva et al. (2013) and Messa and Malavasi (2014). Sundaresan et al. (2003) outlined that new experiments and/or analyses are needed to cast light on the important phenomena that cause turbulence damping or generation. It has a special importance in case of slurry flows. The authors suggested that the experiments should be conducted in simple turbulent flows such as grid turbulence, fully developed pipe or channel flow, or simple axially symmetrical flows. Regardless of geometry, experiments must include a wide range of particle parameters in a single fixed facility.

The paper deals with non-isothermal solid-liquid turbulent flow in horizontal pipeline. The slurry contains fine solid particles of averaged diameters below 10 μm surrounded by water as a carrier liquid. As mentioned above, it is quite common that such slurries exhibit non-Newtonian behaviour. Mathematical modelling of such turbulent flow requires the momentum and energy equations, an equation or equations to calculate the turbulence stress tensor, and rheological model with the yield stress in order to calculate the apparent viscosity. Additionally, the mathematical model requires proper defined turbulence damping function, called also the wall damping function, which is relevant for enhanced turbulence damping, which exists near a pipe wall in such slurries.

The main objective of the paper is to examine the influence of solids concentration on heat transfer coefficient by taking into account the mathematical model with especially designed turbulence damping function.

Numerical predictions demonstrate substantial influence of solids concentration on temperature distribution and heat transfer coefficient.

2. THE PHYSICAL AND THE MATHEMATICAL MODEL

The physical model assumes fine–dispersive slurry, which exhibits a yield stress. The slurry consists of water and solid particles with density of 2500 kg/m^3. The solids concentration by volume varies from C_V=0% to C_V=30%. It is assumed that slurry viscosity is described by apparent viscosity, which can be assigned by the Bingham rheological model. The apparent viscosity and slurry density are constant across the pipe for isothermal flow and dependent on temperature for non-isothermal flow. The flow in horizontal pipe is homogeneous, axially symmetrical, fully developed and turbulent.

Taking into account the aforementioned physical model, the time-averaged momentum equation in cylindrical co-ordinates can be described as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu_{ap}\frac{\partial \bar{U}}{\partial r}-\bar{\rho}\,\overline{u'v'}\right)\right]=\frac{\partial \bar{p}}{\partial x} \tag{1}$$

The turbulent stress component in equation (1) is designated by the Boussinesque hypothesis, as follows:

$$-\rho\overline{u'v'}=\mu_t\frac{\partial \bar{U}}{\partial r} \tag{2}$$

The turbulent viscosity (μ_t), stated in equation (2), is designated with the support of dimensionless analysis, as follows, (Launder and Sharma, 1974):

$$\mu_t=f_\mu\frac{\bar{\rho}k^2}{\varepsilon} \tag{3}$$

The kinetic energy of turbulence (k) and its dissipation rate (ε), which appear in equation (3), are delivered from the Navier-Stokes equations. Earlier research proved that the Launder and Sharma (1974) turbulence model has a potential to predict a slurry flow, (Bartosik and Shook, 1991), therefore this turbulence model was chosen for further development. The final form of k and ε equations for the aforementioned assumptions are the following:

$$\frac{1}{r}\left[r\left(\mu_{ap}+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right]+\mu_t\left(\frac{\partial \bar{U}}{\partial r}\right)^2=\bar{\rho}\varepsilon+2\mu_{ap}\left(\frac{\partial k^{1/2}}{\partial r}\right)^2 \tag{4}$$

$$\frac{1}{r}\left[r\left(\mu_{ap}+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r}\right]+C_1\frac{\varepsilon}{k}\mu_t\left(\frac{\partial \bar{U}}{\partial r}\right)^2=C_2\left[1-0.3\exp\left(-Re_t^2\right)\right]\frac{\bar{\rho}\varepsilon^2}{k}-2\frac{\mu_{ap}}{\bar{\rho}}\mu_t\left(\frac{\partial^2\bar{U}}{\partial r^2}\right)^2 \tag{5}$$

The turbulent Reynolds number in equation (5), was defined by Launder and Sharma (1974) using dimensionless analysis, as follows:

$$\mathbf{Re_t} = \frac{\bar{\rho}\,\mathbf{k}^2}{\mu_{ap}\,\varepsilon} \tag{6}$$

The crucial point of the mathematical model is the proper determination of turbulence damping function (f_μ), which exists in equation (3). Wilson and Thomas (1985) reasoned that in fine-dispersive slurry flow a region close to a pipe wall exhibits increased viscous sub-layer. Therefore the turbulence damping function (f_μ), which is an empirical function, was redesigned in order to predict enhanced damping of turbulence near a pipe wall. The especially designed turbulence damping function, which includes dimensionless yield stress, is described by the following equation (Bartosik, 1997):

$$\mathbf{f_\mu = 0,09\exp\left[\frac{-3,4\left(1+\frac{\tau_o}{\tau_w}\right)}{\left(1+\frac{Re_t}{50}\right)^2}\right]} \tag{7}$$

while the standard turbulence damping function (f_μ), proposed by Launder and Sharma (1974), is the following:

$$\mathbf{f_\mu = 0,09\exp\left[\frac{-3,4}{\left(1+\frac{Re_t}{50}\right)^2}\right]} \tag{8}$$

The new turbulence damping function (7), compared to the standard one (8), demonstrates enhanced turbulence damping. The new turbulence damping function includes dimensionless yield stress (τ_o/τ_w) and has been successfully examined in a comprehensive range of rheological parameters and flow conditions, (Bartosik, 2009, 2010).

Taking into account the Bingham model the apparent viscosity can be defined as follows:

$$\mu_{ap} = \frac{\mu_{PL}}{\left(1-\frac{\tau_o}{\tau_w}\right)} \tag{9}$$

The wall shear stress, which appears in equation (9), is designated from the balance of forces acting on the unit pipe length, so the wall shear stress can be calculated as follows:

$$\tau_w = \frac{\mathbf{dp}}{\mathbf{dx}}\frac{\mathbf{D}}{\mathbf{4}} \tag{10}$$

For isothermal fine-dispersive slurry flow, the mathematical model comprises three partial differential equations, namely (1), (4) and (5), together with the complimentary equations (2), (3), (6), (7), (9), (10).

In order to examine the influence of solids concentration on heat exchange process, the mathematical model is extended by the following energy equation, set up for the temperature:

$$\bar{\rho}\bar{U}\frac{\partial\bar{T}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\mu_{ap}}{Pr} + \frac{\mu_t}{Pr_t}\right)\frac{\partial\bar{T}}{\partial r}\right] \tag{11}$$

Several researchers have been examined intensively the Turbulent Prandtl number, which exists in equation (11). Their studies indicated that for the flow on a plate, the turbulent Prandtl number is about Pr_t=0.5, while for the boundary layer $Pr_t \approx 0.9$, (Blom, 1970).

Convective term in energy equation (11) is determined from the energy balance acting on the unit pipe length (Δx=1m), assuming that the temperature in ox direction varies linearly. The final form of the temperature gradient in ox direction is the following:

$$\frac{\partial\bar{T}}{\partial x} = \frac{2\,q}{\rho_b \bar{U}_b \left(c_p\right)_b R^2} \tag{12}$$

while the Prandtl number is calculated using the apparent viscosity:

$$Pr = \frac{\mu_{ap} c_P}{\lambda} \tag{13}$$

Finally, the mathematical model comprises four partial differential equations, namely momentum and energy equations, and equations for kinetic energy of turbulence and its dissipation rate. Partial differential equations, namely (1), (4), (5) and (11), together with complimentary equations (2), (3), (6), (7), (9), (10), (12) and (13), were solved by finite difference scheme using own computer code. The mathematical model is suitable to predict velocity distribution, frictional head loss, temperature distribution, Nusselt number and heat transfer coefficient of fine-dispersive slurry with a yield stress in horizontal pipelines.

Numerical calculations were performed for known dp/dx. The turbulence constants in the turbulence model are the same as those in the turbulence model of Launder and Sharma (1974), and equal: C_1=1.44; C_2=1.92; σ_k=1.0; σ_ε=1.3, Pr_t=0.9. The mathematical model assumes non-slip velocity at the pipe wall, i.e. U=0, k=0 and ε=0. Axially symmetrical conditions were applied at the centre of the pipe, therefore dU/dr=0, dT/dr=0, dk/dr=0 and dε/dr=0. A differential grid of 80 nodal points distributed on the radius of the pipe has been used. The majority of the nodal points were localized in close vicinity of the pipe wall to ensure the convergence process. The number of nodal points was set up experimentally to ensure nodally independent computations.

3. NUMERICAL PREDICTIONS

In order to perform numerical prediction of influence of solids concentration on the heat transfer coefficient in turbulent slurry flow, it was essential to establish empirical relations of $\tau_o=f(C_V)$ and $\mu_{PL}=f(C_V)$ in order to have physical properties of real slurry flow. It was assumed that the influence of temperature on slurry properties, like slurry density and slurry apparent viscosity, is qualitatively the same as for carrier liquid. However, in the case of specific heat at constant pressure it was assumed that it is the same as for carrier liquid (c_P=4178 J/(kg K). This is not quite right. However, such assumptions are reasonable when one examines the qualitative influence of the solids concentration on slurry temperature distribution.

Numerical simulations of non-isothermal turbulent flow of fine–dispersive slurry with mean particle diameter below 10 μm were made for the pipe with inner diameter D=0.075 m. Solid particles density was 2500 kg/m^3 and solids concentration by volume varied from 0% to 30%. The Reynolds number was defined in accordance with the apparent viscosity concept as follows:

$$\mathbf{Re}_{ap} = \frac{\rho_m \left(U_b\right)_m D}{\mu_{ap}} \tag{14}$$

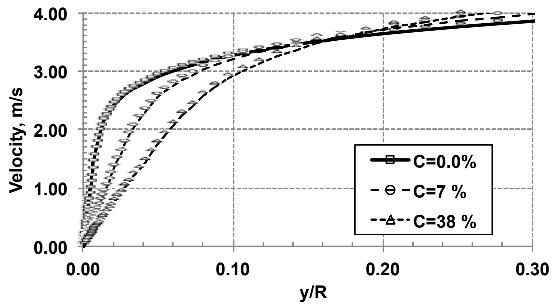

Fig. 1. Dependence of the solids concentration on velocity distribution close to the pipe wall for water and Bingham slurry at constant bulk velocity $(U_b)_m$=4.30 m/s, (isothermal flow)

Predictions of isothermal slurry velocity profiles with constant bulk velocity equal to 4.3 m/s and for solids concentration equal to 7% and 38% by volume are demonstrated in Fig. 1. It is seen in Fig. 1 that there is substantial qualitative and quantitative difference between slurry and water velocity profiles. The decrease of local slurry velocity close to the pipe wall is compensated by the increase of local velocity in the core region. Such significant differences of velocity shapes close to the pipe wall affect the heat transfer process in a slurry flow.

In numerical computations for non-isothermal flow it was assumed that the wall temperature is constant and equal to 293.15 K. The heat flux, acting on unit length of the pipe was applied and equals to q = -200 W/m. Numerical predictions were made for fine-dispersive slurry with solids concentration equal C_V=7%, 20% and 30%. Density and rheological properties of such Bingham slurry are stated in Table 1 for T=293K.

Table 1. Rheological properties of fine-dispersive Bingham slurry

C_V %	ρ_m kg/m^3	τ_o N/m^2	μ_{PL} Pa s
7%	1105.30	4.920	4.33 10^{-3}
20%	1298.56	6.292	7.63 10^{-3}
30%	1448.74	7.791	12.39 10^{-3}

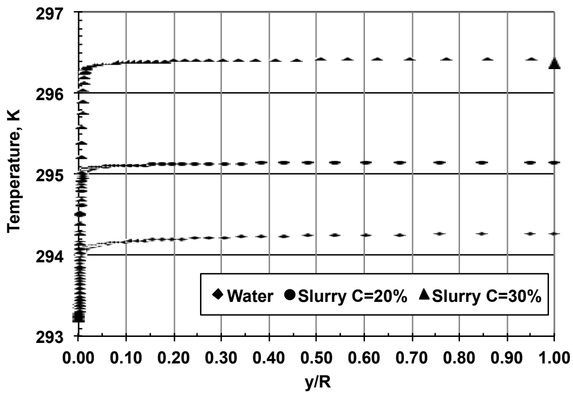

Fig. 2. Temperature distribution in Bingham slurry flow at constant bulk velocity $(U_b)_m$=3.7 m/s, D=0.075 m.

Numerical predictions confirmed the substantial influence of solids concentration on heat exchange in slurry flow. In Fig. 2 the temperature distribution is presented both for slurry, and for water flow in the pipe with inner diameter D=0.075m. Fig. 2 demonstrates that increasing solids concentration causes the increase of temperature difference $\Delta T=T_b-T_w$. Predictions confirmed that even small changes in velocity distribution significantly affect temperature distribution, as shown in Fig. 1 and Fig. 2.

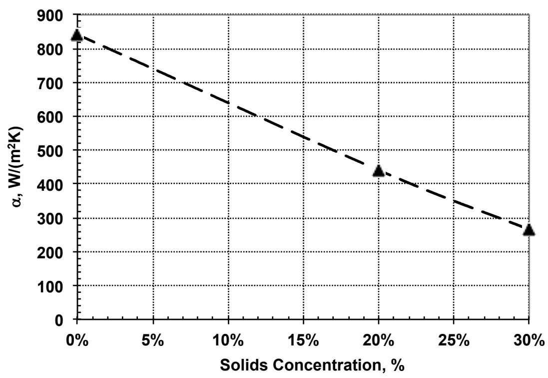

Fig. 3. Dependence of heat transfer coefficient on solids concentration D=0.075 m, Q= -200 W/m.

Increased damping of turbulence, which takes place at the pipe wall, causes the reduction of the heat transfer coefficient (α). Lower heat transfer coefficient means that for the same heat flux acting on the unit pipe length of radius R, and for the same boundary conditions, there is a higher difference of $\Delta T=T_b-T_w$. Taking into account numerical predictions of heat exchange in Bingham slurry flow, it is shown in Fig. 3 that the heat transfer coefficient decreases when the solids concentration increases. Such conclusion is limited to the condition that the wall temperature and the heat flux are constant. The mathematical model was validated previously only for isothermal slurry flow as for non-isothermal flow there are no such experimental data in literature.

4. CONCLUSIONS

Numerical predictions of Bingham slurry, performed for turbulent flow conditions, exhibit the substantial influence of solids concentration on quality and quantity of the heat exchange process. The paper demonstrates substantial effect of the velocity profile on temperature distribution and as a consequence on heat transfer coefficient.

Numerical simulations of solids concentration influence on heat exchange process in turbulent flow of Bingham slurry allow for formulating the following conclusions:

1. The velocity profile at the pipe wall becomes less steep compared to a single-phase flow. This is due to viscous forces, which depend on the apparent viscosity and the damping of turbulence, which exists at the pipe wall.
2. The solids concentration influences the velocity profiles strongly at the pipe wall and as a consequence influences the heat transfer process resulting in the decrease of the heat transfer coefficient with the increase in solids concentration.
3. A less steep slurry velocity profile at the pipe wall results in a decreased heat transfer coefficient.

Possible cause of '*damping of turbulence*" could be the influence of the solid particles on decreasing time interval of 'bursting phenomena' as particles reduce higher order fluctuations. Additional possible reason of existence of damping of turbulence at a pipe wall could be the '*lift forces*'. As a result of lift forces larger particles are pushed away from the pipe wall and are replaced by finer particles, enhancing the viscous forces of the slurry in vicinity of the pipe wall. If the viscous forces are increasing the '*laminarisation*' of the flow takes place (Bartosik, 2008).

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